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Aerodynamic Design Criteria for Supersonic Climb-Cruise Missiles

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An analytical technique is presented which establishes the relationship between aerodynamic characteristics and total mission performance for ramjet-powered missiles which are launched at low altitude and must climb to high altitude for a long-range cruise at supersonic speeds. The analytical expressions obtained can be used to determine desirable aerodynamic characteristics and their variation with velocity given missile propulsion and design parameters such as specific fuel consumption, thrust coefficient, planform area, nozzle exit area, and weight. The expressions are derived for Rutowski minimum fuel-to-climb profiles and optimum specific fuel consumption cruise conditions. The low-altitude segment of the climb is shown to be dominated by the drag coefficient $C_{D_{min}}$ and its variation with velocity. At high altitudes during the climb, the induced drag factor K becomes important. Therefore, reducing $C_{D_{min}}$ at low supersonic speeds and K at high supersonic speeds reduces fuel consumption during the climb. The desirable lift coefficient at minimum drag C_{L_0} is shown to be small at low speeds and to increase with velocity. The ideal climb-cruise configuration will have a $C_{D_{min}}$ and K which are low at low speeds and decrease strongly with increasing speed. The C_{L_0} will also be low at low speed but increase greatly with increasing speed. These results and the analytical expressions derived can be used by the missile designer as aerodynamic design criteria during configuration development.

Nomenclature

a	= aerodynamic load factor
A	= q^2 coefficient of climb equation
A_5	= nozzle throat area
A_6	= nozzle exit area
A_6/A_5	= nozzle expansion ratio
B	= q coefficient of climb equation
C	= constant coefficient of climb equation
$C_{D_{min}}$	= minimum drag coefficient
C_L	= lift coefficient = L/qS
C_{L_0}	= lift coefficient at minimum drag
C_T	= thrust coefficient = T/qA_5
D	= drag
E/W	= specific energy = $h + V^2/2g$
g	= gravitational constant
h	= altitude
K	= induced drag factor
L	= lift
q	= dynamic pressure
S	= reference area = planform area
SFC	= specific fuel consumption
SRG	= specific range
T	= thrust
V	= velocity
W	= weight
β	= atmospheric density gradient divided by density
θ	= $V^2\beta/g$
$\partial/\partial V$	= partial derivative with respect to velocity
$\left.\frac{\partial}{\partial V}\right)_{E/W}$	= partial derivative with respect to velocity for constant specific energy

Introduction

OFTEN the missile designer is faced with the question of trading climb performance for improved cruise performance. For example, the addition of wings can improve cruise capability, but wings also increase both drag and fuel consumption during the climb. The additional fuel used in the climb may offset the fuel saved by the more efficient cruise.

Traditionally, these typical design questions have been answered by establishing a configuration, determining its aerodynamic and propulsion characteristics, and simulating the trajectory with a three degree-of-freedom trajectory computer code. Another configuration is then designed or the aerodynamic characteristics of the original are perturbed and a new trajectory simulated. The resulting performance is compared and used as a basis for reconfiguring.

Unfortunately, the results using this approach are often ambiguous and obscure. For example, the payoff parameter selected for climb performance evaluation is often fuel consumed during climb. Computer sensitivity studies for fixed-geometry ramjet engines often indicate very nonlinear sensitivities of climb fuel to aerodynamic coefficient changes. The fixed-engine ramjet cycle performance characteristics dominate the sensitivity results, rather than the aerodynamic perturbations. Inconsistent results such as these can be eliminated by using a simplified propulsion model and an energy approach to climb-sensitivity analysis.

During Phase I of the Aerodynamic Configured Missile (ACM) Development Program[1] the desirable aerodynamic characteristics for ramjet-powered, supersonic cruise missiles were established using an energy approach based upon the formulation by Rutowski[2] for the general aircraft performance problem. Although "Rutowski" flight profiles often are impossible to follow with a real missile, they do represent the best an ideal missile could do and the climb performance is sensitive to aerodynamic characteristics. The results can also be coupled with optimum cruise solutions to give an optimum total mission profile and its sensitivity to aerodynamic changes.

For a powered vehicle, the higher the energy requirements to overcome climb drag, the more important improving climb aerodynamics is. For a short-range air-to-air mission, kinetic energy dominates the mission. On the other hand, for a long-range high L/D configuration cruise energy requirements dominate. However, for many missions, both climb and cruise energy requirements are not negligible. Therefore, reducing climb energy requirements by more climb efficient aerodynamic design can often improve mission performance.

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During climbs the altitude, velocity, thrust, and specific fuel consumption change and closed-form climb trajectory solutions are not possible except over small segments of the trajectory[3] or where acceleration is negligible.[4] However, if missile weight change is small, the throttle setting fixed, and the aerodynamic load factor constant, three aerodynamic and two propulsion parameters affect the minimum-fuel climb profile. The aerodynamic parameters are $C_{D_{\min}}$, KC_{L_0} and the ramjet propulsion parameters are C_TA_5 and SFC. Given these parameters as a function of velocity analytical expressions for minimum-fuel climb conditions can be established. Although closed-form solutions are still not possible, these expressions give insight into the desirable aerodynamic characteristics for climb-cruise missiles.

Derivation of Climb Equations

General Relations

The Rutowski relation for minimum fuel climb is given by

$$\frac{\partial}{\partial V}\left[\frac{(T-D)V}{WT\mathrm{SFC}}\right]_{E/W}=0.0 \tag{1}$$

assuming $W=$ constant and

$$T=qC_TA_5 \tag{2}$$

$$D=qS[C_{D_{\min}}+K(C_L-C_{L_0})^2] \tag{3}$$

$$C_L=aW/qS \tag{4}$$

and C_TA_5, SFC, $C_{D_{\min}}$, K, and C_{L_0} are known functions of velocity and altitude.

The expanded form of Eq. (1) becomes

$$\left\{\frac{\partial T}{\partial V}-\frac{\partial D}{\partial V}+(T-D)\left[\frac{1}{V}-\frac{1}{T}\frac{\partial T}{\partial V}-\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}\right]\right\}_{E/W}=0.0 \tag{5}$$

Each term in Eq. (5) is then computed by substituting Eqs. (2-4) into Eq. (5) and performing the indicated differentiation. Because the differentiation is performed at constant specific energy (E/W), the dynamic pressure derivatives have a special form which becomes

$$\left.\frac{\partial q}{\partial V}\right)_{E/W}=\frac{q}{V}(2-\theta) \tag{6}$$

where

$$\theta=(V^2\beta)/g \tag{7}$$

With this substitution the derivatives of Eq. (5) become

$$\left.\frac{1}{T}\frac{\partial T}{\partial V}\right)_{E/W}=\frac{1}{V}(2-\theta)+\frac{1}{C_TA_5}\frac{\partial C_TA_5}{\partial V} \tag{8}$$

$$\begin{aligned}\frac{\partial D}{\partial V}&=S[C_{D_{\min}}+K(C_L-C_{L_0})^2]\left[\frac{q}{V}(2-\theta)\right]\\&+qS\left[\left.\frac{\partial C_{D_{\min}}}{\partial V}\right)_{E/W}+\left.\frac{\partial K}{\partial V}\right)_{E/W}(C_L-C_{L_0})^2\right.\\&\left.+2K(C_L-C_{L_0})\left(\frac{\partial C_L}{\partial V}-\frac{\partial C_{L_0}}{\partial V}\right)_{E/W}\right]\end{aligned} \tag{9}$$

where

$$\left.\frac{\partial C_L}{\partial V}\right)_{E/W}=-\frac{aW}{qS}\frac{(2-\theta)}{V} \tag{10}$$

Equations (8-10) can be substituted into Eq. (5) and after manipulation, a quadratic equation for dynamic pressure of the form

$$Aq^2+Bq+C=0.0 \tag{11}$$

can be obtained, where

$$\begin{aligned}A&=C_TA_5\left[\frac{1}{V}-\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}\right]+C_{D_{\min}}S\left[-\frac{1}{V}+\frac{1}{C_TA_5}\frac{\partial C_TA_5}{\partial V}\right.\\&\left.+\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}-\frac{1}{C_{D_{\min}}}\frac{\partial C_{D_{\min}}}{\partial V}\right]\\&+KC_{L_0}^2S\left[-\frac{1}{V}+\frac{1}{C_TA_5}\frac{\partial C_TA_5}{\partial V}+\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}-\frac{1}{K}\frac{\partial K}{\partial V}\right]\\&-2KC_{L_0}S\frac{\partial C_{L_0}}{\partial V}\end{aligned} \tag{12}$$

$$\begin{aligned}B&=2K\frac{aW}{S}C_{L_0}S\left[\frac{1}{V}-\frac{1}{C_TA_5}\frac{\partial C_TA_5}{\partial V}-\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}\right.\\&\left.-\frac{(2-\theta)}{V}+\frac{1}{K}\frac{\partial K}{\partial V}\right]+2KaW\frac{\partial C_{L_0}}{\partial V}\end{aligned} \tag{13}$$

$$\begin{aligned}C&=\frac{Ka^2W^2}{S^2}S\left[-\frac{1}{V}+\frac{1}{C_TA_5}\frac{\partial C_TA_5}{\partial V}+\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}\right.\\&\left.+\frac{2(2-\theta)}{V}-\frac{1}{K}\frac{\partial K}{\partial V}\right]\end{aligned} \tag{14}$$

Note that the derivatives of SFC, C_TA_5, $C_{D_{\min}}$, K, and C_{L_0} appear throughout the equations. If A, B, and C are weak functions of density, then at a given velocity the coefficients of Eq. (11) are constant and it can be solved for dynamic pressure. This dynamic pressure corresponds to an altitude-velocity point on the minimum-fuel climb profile. Equations (12-14) identify the parameters important to the climb. For example, C_TA_5 and $C_{D_{\min}}S$ are important in the q^2 coefficient, $C_{L_0}S$ in the q coefficient, and KS in the constant term.

Typical Magnitudes of Aerodynamic Terms

The relative magnitudes of these terms is best addressed by considering wind-tunnel data for a variety of configurations. Figure 1 shows the geometry of configurations which were tested as part of the ACM Phase I wind-tunnel test program.[5] Configurations consist of a half-ellipse noncircular body, triangular cross-section lifting body, blended wing-body, and rectangular cross-section wing-body. Data were obtained at Mach 3, 4, and 5. These wind-tunnel data were curve fit by a parabolic curve of the form $C_D=C_{D_{\min}}+K\ (C_L-C_{L_0})^2$. The parabolic assumption provided curve-fit errors of less than 1% at lift coefficients of 0.10 or less. The parameters $C_{D_{\min}}$, K, and C_{L_0} at Mach 3, 4, and 5 were then curve fit with a power law curve as a function of Mach number. Typical missile data similar to that tested by Sawyer and Hayes[6] were also curve fit for comparison purposes since the ACM models represent advanced supersonic cruise concepts. Figures 2-4 present the results of these curve fits. Although the $C_{D_{\min}}$ magnitude shown in Fig. 2 varies significantly for each configuration, the variation with Mach number is similar. The induced drag factor K presented in Fig. 3 shows sensitivity to both configuration and Mach number. The lifting body (square symbols) exhibits a weak Mach number dependence compared to the others. The C_{L_0} variation with Mach number is large and not consistent as shown in Fig. 4. Some of this variation is the result of the curve-fit accuracy. The parabolic curve fit is nearly vertical at $C_{D_{\min}}$ and a slight error in $C_{D_{\min}}$ results in a large C_{L_0} error.

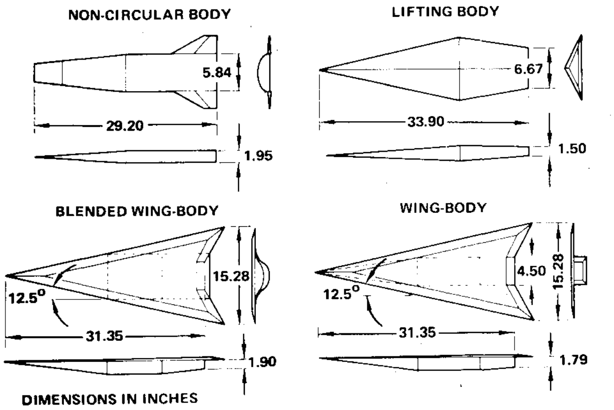

Fig. 1 Advanced configuration concepts have been tested.

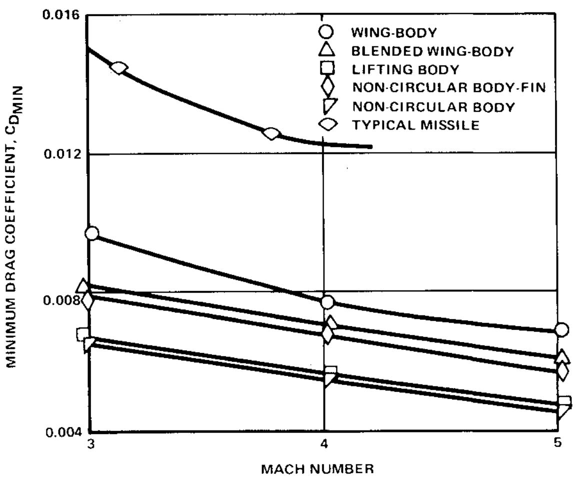

Fig. 2 $C_{D_{\min}}$ varies significantly for different configurations.

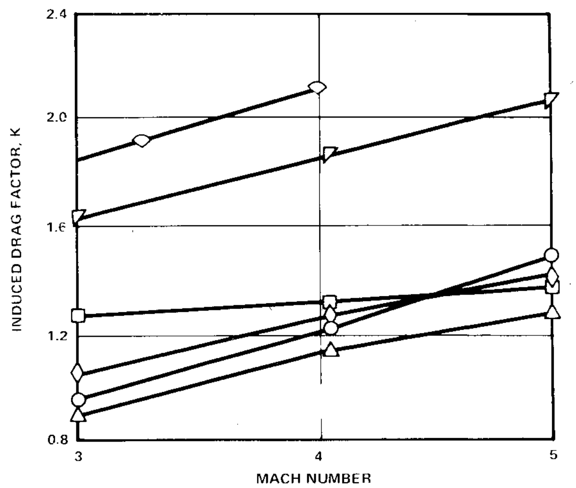

Fig. 3 Induced drag factor Mach dependency is configuration dependent.

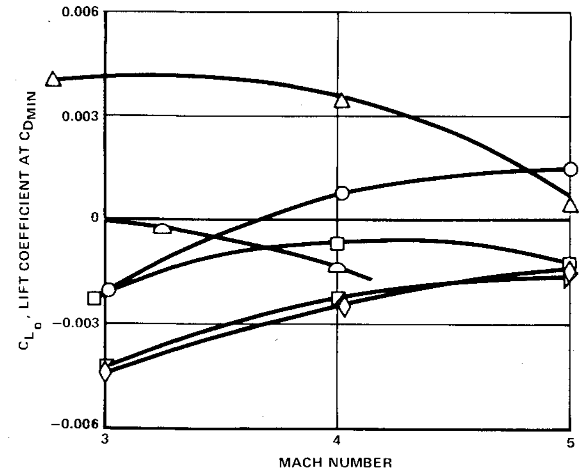

Fig. 4 C_{L_0} shows a strong Mach dependency.

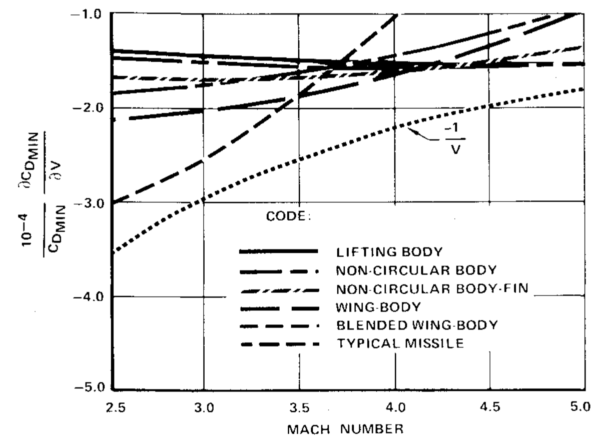

Fig. 5 $C_{D_{\min}}$ derivative term is large for typical missile.

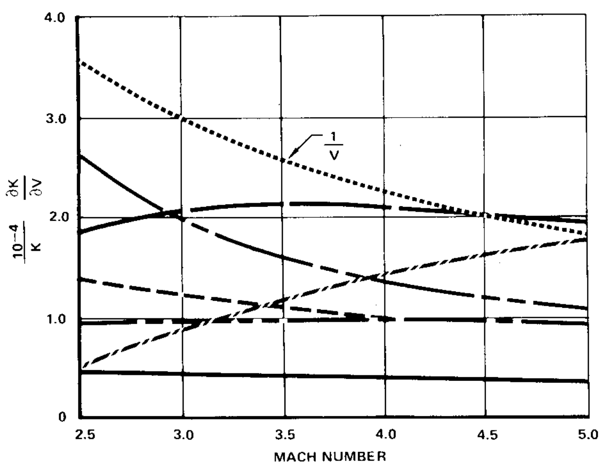

Fig. 6 K derivative term is large for winged concepts.

Figures 5 and 6 present the variation of the derivative terms

$$\frac{1}{C_{D_{\min}}} \frac{\partial C_{D_{\min}}}{\partial V} \quad \text{and} \quad \frac{1}{K}\frac{\partial K}{\partial V}$$

which are terms identified as important to the climb in Eqs. (12-14). (They are plotted vs sea level Mach number as are the results shown in subsequent figures.) Although the variation with Mach number is not as great as $C_{D_{\min}}$ and K in Figs. 2 and 3, configuration differences still exist and may be important to the climb performance depending upon the magnitude of the propulsion terms.

Typical Magnitudes for Propulsion Terms

The above data provided typical magnitudes for the important aerodynamic terms in the climb relations. A similar evaluation of the propulsion terms was accomplished. As mentioned previously, a fixed-geometry ramjet engine imposes considerable contraints on a climb profile. Therefore, a variable geometry inlet and nozzle ramjet engine was modeled

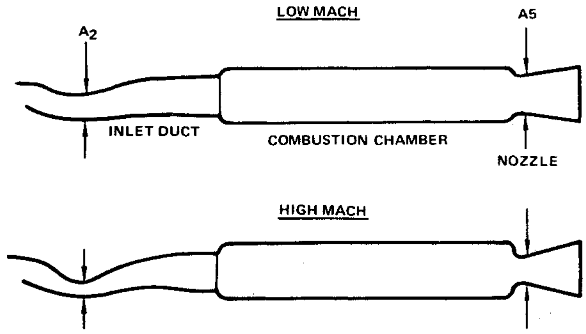

Fig. 7 Variable geometry A_2 and A_5 change with Mach number.

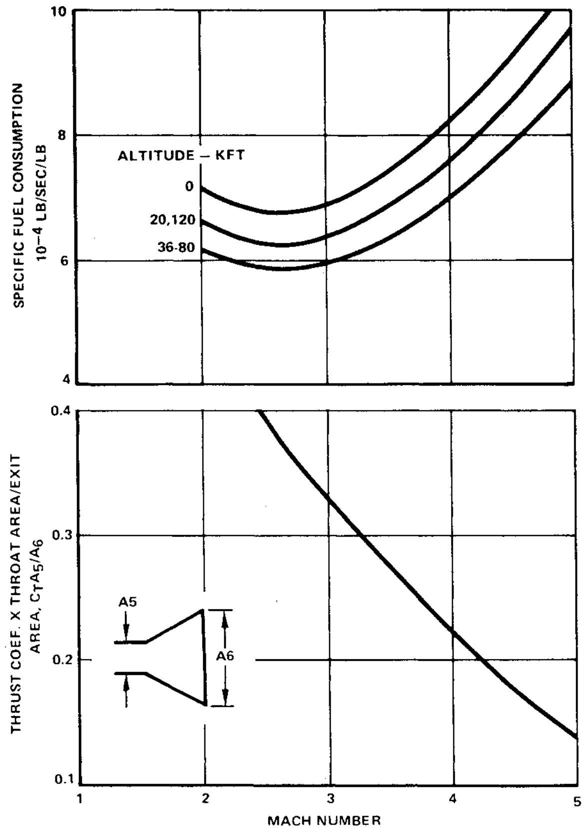

Fig. 8 Variable geometry ramjet propulsion model selected.

and is depicted schematically in Fig. 7. The inlet area and nozzle throat area are varied to achieve the best SFC at each Mach number. For example, at low Mach numbers the nozzle throat area A_5 is large. At Mach numbers near cruise conditions, A_5 is reduced to give an optimum nozzle expansion ratio. Figure 8 gives the SFC and $C_T A_5 / A_6$ variation with Mach and altitude. While SFC is a weak function of altitude, $C_T A_5 / A_6$ is independent of altitude. Although these curves do not represent the very best performance of a ramjet engine (e.g., total pressure recovery could be improved with different designs), they do reflect a high-performance engine. For example, a fixed-geometry engine may have an SFC 20% higher and a $C_T A_5 / A_6$ which is lower and more nearly constant with Mach rather than strongly varying.

Order of Magnitude Analysis

Using curve fits of the above data, magnitudes of terms in Eq. (12) were evaluated. The q^2 coefficient A includes $C_{D_{\min}}$, KC_{L0}^2, and $\partial C_{L0}/\partial V$ terms which are functions only of velocity. For a wide range of configurations the $C_{D_{\min}}$ term dominates the coefficient as shown in Figs. 9 and 10. It is usually greater than the other terms by a factor of 100.

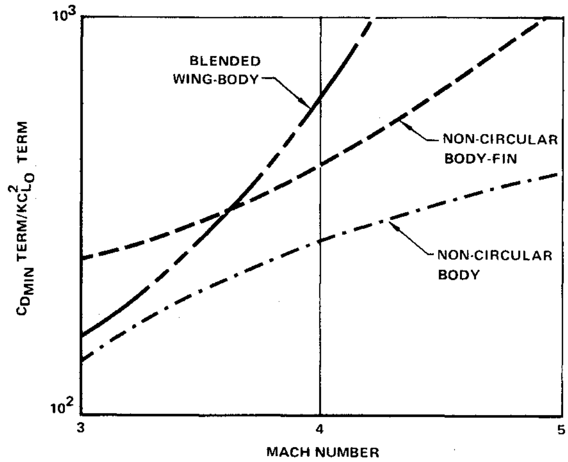

Fig. 9 $C_{D_{\min}}$ term is much larger than KC_{L0}^2 term.

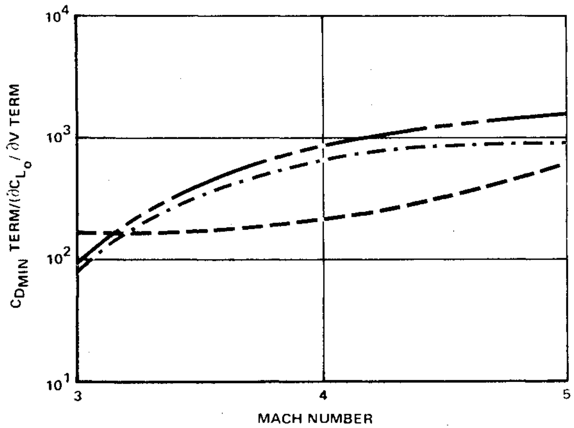

Fig. 10 $C_{D_{\min}}$ term is much larger than $\partial C_{L0}/\partial V$ term.

Therefore, for most configurations the q^2 coefficient reduces to

$$A \simeq C_T A_5 \left[\frac{1}{V} - \frac{1}{\text{SFC}} \frac{\partial \text{SFC}}{\partial V} \right] + C_{D_{\min}} S \left[-\frac{1}{V} + \frac{1}{C_T A_5} \frac{\partial C_T A_5}{\partial V} + \frac{1}{\text{SFC}} \frac{\partial \text{SFC}}{\partial V} - \frac{1}{C_{D_{\min}}} \frac{\partial C_{D_{\min}}}{\partial V} \right] \tag{15}$$

A further examination of the q term B and constant term C of Eqs. (13) and (14) is shown in Fig. 11 for the noncircular body-fin. (All subsequent data presented will be based upon the aerodynamic data for this configuration only.) Typical magnitudes are plotted as a function of dynamic pressure. The q term is negligible except possibly at low dynamic pressure, the q^2 term dominates at high q, and the constant term becomes important at low q. As a consequence, Eq. (12) reduces to

$$q^2 A = 0.0 \tag{16}$$

at low altitudes and low load factors and

$$q^2 A + C = 0.0 \tag{17}$$

at high altitudes or high load factors.

Therefore, for low-load-factor climbs, the initial climb altitude-velocity profile will be independent of aerodynamic

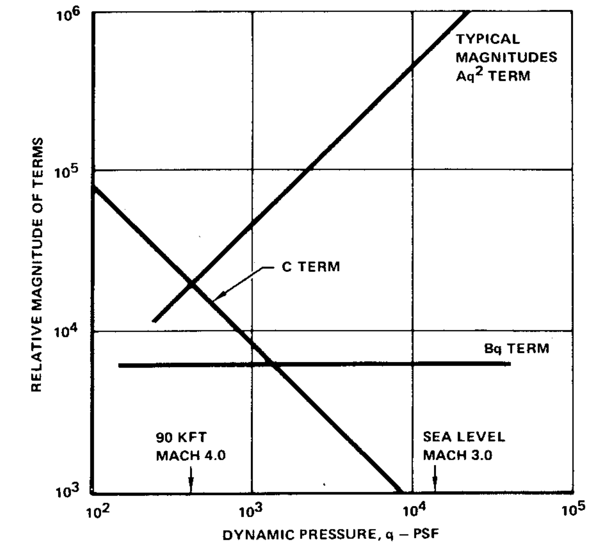

Fig. 11 q^2S **term dominates at low altitude only.**

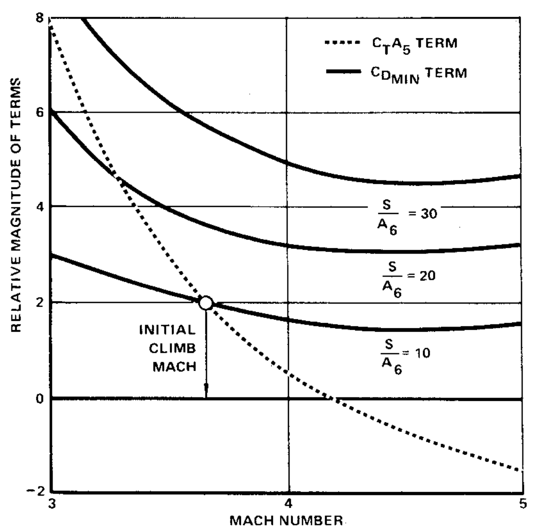

Fig. 12 C_TA_5 **and** $C_{D_{\min}}$ **terms determine initial climb Mach number.**

load factor. In fact, the optimum climb will be characterized by a nearly constant velocity during the initial altitude change portion of the flight. The climb velocity will be given by the solution of the equation.

$$C_TA_5\left[\frac{1}{V}-\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}\right]=-C_{D_{\min}}S\left[-\frac{1}{V}+\frac{1}{C_TA_5}\times\frac{\partial C_TA_5}{\partial V}+\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}-\frac{1}{C_{D_{\min}}}\frac{\partial C_{D_{\min}}}{\partial V}\right] \tag{18}$$

Figure 12 shows typical solutions to this equation for parametric values of the parameters S/A_6. (The nozzle exit area A_6 is introduced here by rewriting C_TA_5 as $C_T(A_5/A_6)A_6$ where A_6/A_5 is the nozzle expansion ratio. This permits a form of the thrust coefficient term consistent with the curve fit in Fig. 8.) The parameter S/A_6 can be interpreted as a measure of the thrust-to-drag ratio of the missile when S is the planform area and A_6 the nozzle exit area, the higher S/A_6 the more planform area per area of nozzle and usually the higher drag. Higher drag results in a lower climb velocity and altitude is achieved with less velocity and more climb fuel used. If S/A_6 becomes large enough there will be no solution for climb velocity within typical operating speeds of ramjet engines.

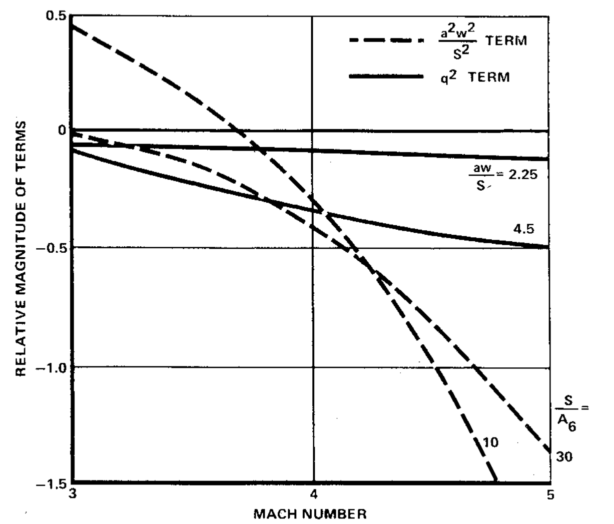

Fig. 13 q^2 **and** a^2W^2/S^2 **terms determine climb load factor.**

The two aerodynamic terms which enter into the estimation of optimum climb velocity are:

$$C_{D_{\min}}S \quad \text{and} \quad \frac{1}{C_{D_{\min}}}\frac{\partial C_{D_{\min}}}{\partial V}$$

A low $C_{D_{\min}}S$ will result in higher climb velocities for the same engine. In addition, a more negative

$$\frac{1}{C_{D_{\min}}}\frac{\partial C_{D_{\min}}}{\partial V}$$

i.e., a more rapid decrease with Mach number, will result in higher climb velocities and more efficient climbs.

At high altitudes or high load factor, Eq. (17) applies. Then the equation which must be solved for the climb path becomes

$$q^2\left\{C_TA_5\left[\frac{1}{V}-\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}\right]+C_{D_{\min}}S\left[-\frac{1}{V}+\frac{1}{C_TA_5}\frac{\partial C_TA_5}{\partial V}+\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}-\frac{1}{C_{D_{\min}}}\frac{\partial C_{D_{\min}}}{\partial V}\right]\right\}$$
$$\approx-\frac{Ka^2W^2S}{S^2}\left[-\frac{1}{V}+\frac{1}{C_TA_5}\frac{\partial C_TA_5}{\partial V}+\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}+\frac{2(2-\theta)}{V}-\frac{1}{K}\frac{\partial K}{\partial V}\right] \tag{19}$$

In addition to the sensitivity to $C_{D_{\min}}$, the induced drag factor K and $[(1/K)(\partial K/\partial V)]$ now enter the equation. The latter term can be neglected in most cases when compared to the $[2(2-\theta)]/V$ term. Figure 13 shows a typical range of values for the right and left sides of Eq. (19). Two design parameters now enter the climb solution. S/A_6 remains as an important parameter as shown in Fig. 12. In addition, the planform loading parameter W/S is important on the right-hand side. The lower the planform loading the less important is the right-hand side of Eq. (19). This is desirable to minimize climb acceleration at high induced drag. Similarly a high planform loading can be offset by a low induced drag factor K. A low value of K results in a climb velocity similar to that of the low-altitude optimum. As in Eq. (18), solutions exist for which there is no reasonable velocity for the propulsion model being considered.

Cruise Design Criteria

Equations (18) and (19) establish the functional dependence of the optimum climb profiles on aerodynamic, propulsion, and design parameters. However, the climb must be related to the cruise conditions before a combined climb-cruise design criteria can be established. The optimum cruise point at a given altitude and weight can be defined as the Mach number with the best specific range, i.e., nautical miles per pound of fuel used,

$$\mathrm{SRG}=\frac{VL}{\mathrm{SFC}\cdot W\cdot D}=\frac{V}{\mathrm{SFC}\cdot D} \tag{20}$$

At the cruise point lift equals weight and, therefore, only V, SFC, and D enter into the determination of the best cruise velocity. This velocity can be determined by differentiating Eq. (20) with respect to velocity and solving for the velocity at which specific range is maximized, i.e.,

$$\frac{\partial \mathrm{SRG}}{\partial V}=0.0=\frac{\mathrm{SRG}}{V}-\frac{\mathrm{SRG}}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}-\frac{\mathrm{SRG}}{D}\frac{\partial D}{\partial V} \tag{21}$$

Rearranging yields,

$$\frac{\partial D}{\partial V}=D\left[\frac{1}{V}-\frac{1}{\mathrm{SFC}}\frac{\partial \mathrm{SFC}}{\partial V}\right] \tag{22}$$

Note the term in brackets is the same term appearing on the thrust side of Eq. (18). Expanding the drag derivative with velocity yields a result similar to Eq. (9).

$$\frac{\partial D}{\partial V}=qS\Big[C_{D_{min}}\Big(\frac{2}{V}+\frac{1}{C_{D_{min}}}\frac{\partial C_{D_{min}}}{\partial V}\Big)+KC_{L0}^2\Big(\frac{2}{V}+\frac{1}{K}\frac{\partial K}{\partial V}\Big)+2KC_{L0}\frac{\partial C_{L0}}{\partial V}\Big]-2WK\Big(\frac{C_{L0}}{K}\frac{\partial K}{\partial V}+\frac{\partial C_{L0}}{\partial V}\Big)+\frac{1}{q}\frac{W^2K}{S}\Big(\frac{1}{K}\frac{\partial K}{\partial V}-\frac{2}{V}\Big) \tag{23}$$

An order-of-magnitude check of this equation indicates that the C_{L0} terms cannot be neglected for a typical case. Good specific range is achieved by having low drag and high velocity as is evident from Eq. (20). In addition, the best cruise velocity for a given propulsion system is sensitive to the $C_{D_{min}}$, K, and C_{L0} velocity derivatives. High-velocity cruise is achievable for high local $C_{D_{min}}$, K, and C_{L0} derivatives. Figure 14 shows the relation between the drag derivative term and the propulsion term at 90 kft altitude. The velocity of optimum specific range corresponds to the crossover velocity of the two curves.

The design parameter of importance is W/S as for the climb. Decreased W/S will result in a higher cruise velocity and better specific range. Increased $C_{D_{min}}$, K, $[(1/C_{D_{min}})(\partial C_{D_{min}}/\partial V)]$, or $[(1/K)(\partial K/\partial V)]$ will result in lower cruise velocities. However, an increased C_{L0} or $\partial C_{L0}/\partial V$ will increase velocity. In fact a $C_{L0}\simeq W/qS$ is the most desirable for the cruise.

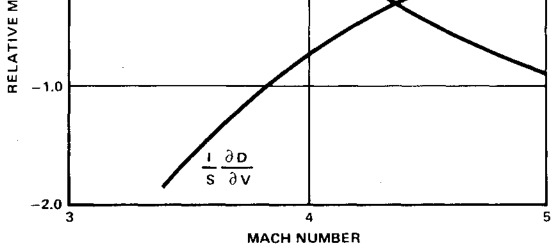

Fig. 14 Optimum cruise Mach is a function of drag and SFC derivatives.

Combined Climb-Cruise Design Criteria

The climb and cruise Eqs. (18), (19), and (23) can be considered together to establish aerodynamic design criteria for climb-cruise missiles. The cruise altitude and velocity are related to the climb requirements as follows. The climb is defined as that portion of the flight which takes the missile from ramjet takeover velocity and altitude to cruise velocity and altitude. The cruise velocity and altitude represent an energy level which the climb must achieve with minimum fuel use.

The optimum climb profile which achieves the cruise altitude and velocity corresponds to a particular aerodynamic load factor. As indicated in Fig. 13, increasing aerodynamic load factor increases the optimum climb velocity. Higher load factor also results in more fuel being used in the climb and a stronger sensitivity to W/S and K.

The initial climb velocity is independent of this load factor as shown in Eq. (18). This velocity is determined by $C_{D_{min}}S$ and $[(1/C_{D_{min}})(\partial C_{D_{min}}/\partial V]$.

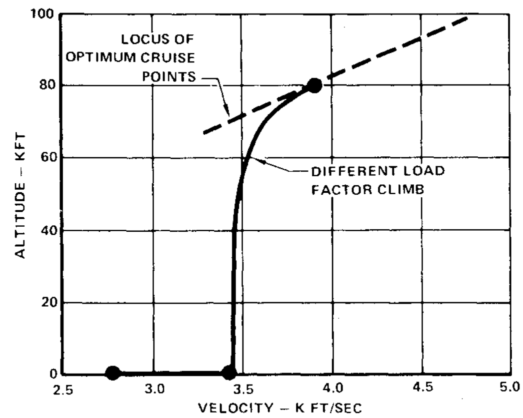

Fig. 15 Climb and cruise requirements are combined.

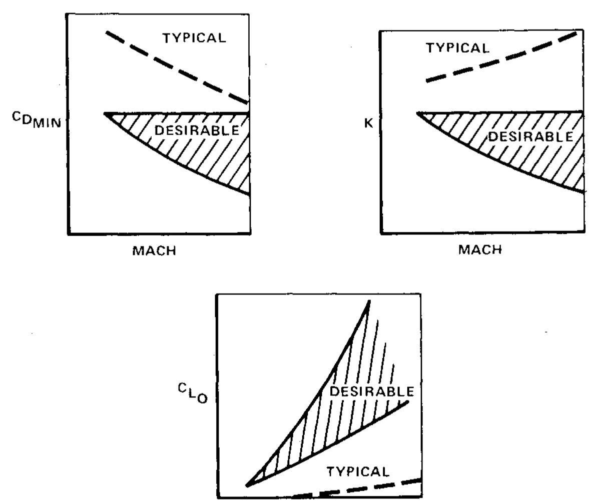

Fig. 16 Desirable and typical aerodynamic characteristics are very different.

These results can be combined as follows. First, Eq. (23) is solved at a given altitude for the optimum cruise velocity. Equation (19) is then solved for the aerodynamic load factor required to achieve the cruise altitude at this velocity. Equation (18) is also solved for the initial climb velocity. The climb profile can then be constructed as shown in Fig. 15. From the ramjet takeover speed, a constant altitude acceleration to the initial climb velocity given by Eq. (18) occurs. The initial altitude change portion of the climb is performed at this velocity until the load factor term of Eq. (19) becomes finite. Then a high-altitude acceleration to the optimum cruise velocity occurs. The locus of optimum cruise points in Fig. 15 represents a contour of optimum cruise altitude-velocity points. Climbs can be generated to each of these points and the best climb-cruise profile selected based upon the total fuel load. The "best" climbers are configurations which have high initial climb velocities and require a low aerodynamic load factor to achieve the cruise altitude-velocity. These configurations climb out of the high-density low altitudes quickly and spend little time accelerating to the cruise velocity at high altitude. In effect, they get to the very efficient cruise condition as quickly as possible.

The best aerodynamics based upon Eqs. (18), (19), and (23) for this type of climb-cruise mission are depicted schematically in Fig. 16. The most desirable drag polars are those which have low $C_{D_{\min}}$, a low C_{L_0} at low Mach, high C_{L_0} at high Mach, and a low K. This leads to a range of desirable coefficient variation with Mach as identified in Fig. 16. A reduced $C_{D\min}$ and K with Mach and an increased C_{L_0} with Mach are desirable but not consistent with a real configuration. In fact, strong C_{L_0} variations are not observed except with favorable interference concepts. However, Fig. 16 does provide the goals for aeroconfiguring a climb-cruise missile.

Conclusions

Minimum energy climb solutions can be used to determine the desirable climb aerodynamic characteristics as a function of missile design parameters, ramjet propulsion parameters, and cruise altitude and velocity. The important design parameters are identified as the planform area divided by nozzle exit area S/A_6 and the planform loading W/S. The relevant propulsion parameters are the thrust coefficient divided by the nozzle expansion ratio and multiplied by nozzle exit area $C_T(A_5/A_6)A_6$, and the derivatives $[(1/C_TA_5)(\partial C_TA_5/\partial V)]$ and $[(1/\mathrm{SFC})(\partial \mathrm{SFC}/\partial V)]$.

The two major aerodynamic characteristics affecting fuel used in a typical low aerodynamic load factor climb are $C_{D\min}S$ and $[(1/C_{D\min})(\partial C_{D_{\min}}/\partial V)]$. Fuel used in the climb to a cruise velocity is reduced if $C_{D\min}S$ is small and the derivative term is strongly negative. At high altitudes or high load factor the climb performance becomes sensitive to the planform loading W/S and the induced drag factor multiplied by planform area KS. The cruise altitude and velocity are achieved with less fuel used if W/S or KS are reduced.

In general, K and W/S are reduced by increasing either wing or body planform area. $C_{D_{\min}}$ is reduced by nose shape changes and fineness ratio increases. However, $C_{D_{\min}}S$ and KS can increase with increased planform. Therefore, a trade between increased planform to improve cruise performance and the resulting decrease in climb performance is indicated.

A low C_{L_0} during the initial climb and an increasing value at high speeds is desirable for both improved climb and cruise performance.

Finally, the results for initial climb velocity, climb aerodynamic load factor, and cruise velocity can be used for preliminary evaluation of configuration performance. The "good" climbers have a high climb velocity, a low aerodynamic load factor, and high cruise velocity.

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